SIMULATION OF TEMPERATURE DISTRIBUTIONS IN VACUUM

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UDC 536.21:533.59

A method is presented for simulating the temperature distribution in an object when there is no convective heat transfer. Similarity of boundary conditions is ensured by a heat shield with a controlled temperature serving as a free parameter.

High-vacuum studies of temperature distributions in small-scale objects containing fuel elements are difficult because the introduction of temperature-sensitive elements distorts the temperature distribution being studied. It becomes necessary to produce large-scale models having a temperature distribution similar to that under study.

The model must be geometrically similar to the object, and the ratio of physical constants in the various regions of the model must be the same as in the object [1].

In the case under discussion the physical constants are the surface emissivity ε and the thermal conductivity λ.

Thus the requirement of constant ratios of physical constants has the form

$$\frac{\lambda_{i\,\mathrm{ob}}}{\lambda_{(i+1)\mathrm{ob}}}=\frac{\lambda_{i\mathrm{a}}}{\lambda_{(i+1)\mathrm{a}}};\quad \frac{\varepsilon_{i\mathrm{ob}}}{\varepsilon_{(i+1)\mathrm{ob}}}=\frac{\varepsilon_{i\mathrm{a}}}{\varepsilon_{(i+1)\mathrm{a}}},\tag{1}$$

where λ_{iob} and λ_{ia} are the thermal conductivities of the i-th region of the object and the model respectively; $\varepsilon_{\mathrm{iob}}$ and $\varepsilon_{\mathrm{ia}}$ are the surface emissivities of the i-th region of the object and model.

In addition the boundary conditions on the surface of the model must be similar to those on the surface of the object. This requires preserving the Biot number

$$\frac{\alpha_{\mathrm{ia}}l_{\mathrm{ia}}}{\lambda_{i\mathrm{a}}}=\frac{\alpha_{\mathrm{iob}}\,l_{\mathrm{iob}}}{\lambda_{i\mathrm{ob}}}.\tag{2}$$

Here α_{iob} and α_{ia} are the radiative heat-transfer coefficients for the i-th region of the object and the model, and l_{iob} and l_{ia} are characteristic linear dimensions of the i-th region of the object and model.

It is clear from (2) that a model having linear dimensions c times as large as those of the object $l_{\mathrm{ia}}/l_{\mathrm{iob}} = c$, must be made of materials having thermal conductivities c times as large as those of the corresponding regions of the object:

$$\lambda_{i\mathrm{a}}=c\lambda_{i\mathrm{ob}}\tag{3}$$

or the surfaces of the corresponding regions of the object and model must have emissivities $\varepsilon_{\mathrm{iob}}$ and $\varepsilon_{\mathrm{ia}}$ such that

$$\alpha_{i\mathrm{a}}=\frac{1}{c}\,\alpha_{i\mathrm{ob}}.\tag{4}$$

In this case identical temperatures at corresponding points of the object and model are achieved by a change in the dissipated power P, so that the dimensionless combination $P/l_i\lambda_i T_i$ remains unchanged.

Thus the simulation of the temperature distribution in a vacuum requires only the geometric similarity of the model and object. The construction of a model corresponding to requirements (3) or (4) does not represent a practical possibility.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 20, No. 5, pp. 925-927, May, 1971. Original article submitted April 23, 1970.

We consider the boundary conditions on the surface of the model surrounded by a heat shield [2] with a controlled temperature in various regions:

$$\lambda_{ia} \frac{dT_{ia}}{dn} = -l_{ia}\alpha_{ia}(T_{ia} - T_{is}). \tag{5}$$

The boundary conditions on the surface of the object have the form

$$\lambda_{iob} \frac{dT_{iob}}{dn} = -l_{iob}\alpha_{iob}(T_{iob} - T_0). \tag{6}$$

Here T_{iob} and T_{ia} are the mean surface temperatures of the i-th region of the object and the model; T_0 is the ambient temperature; T_{is} is the temperature of the i-th region of the shield receiving radiation from the corresponding region of the model, $n = z_{ia}/l_{ia} = z_{iob}/l_{iob}$ is a dimensionless vector normal to the surface of the model and the object, where z_i is the normal to the surface.

For similar boundary conditions on the surface of the model and the object, Eqs. (5) and (6) written in terms of relative quantities must be identical. The transformation of (5) and (6) to dimensionless form gives

$$\frac{d\left(\frac{T_{ia}}{T_0}\right)}{dn} = -\frac{\alpha_{ia}l_{ia}}{\lambda_{ia}}\left(\frac{T_{ia} - T_{is}}{T_0}\right) \tag{7}$$

and

$$\frac{d\left(\frac{T_{iob}}{T_0}\right)}{dn} = -\frac{\alpha_{iob}l_{iob}}{\lambda_{iob}}\left(\frac{T_{iob} - T_0}{T_0}\right). \tag{8}$$

Under the condition

$$\frac{\alpha_{iob}l_{iob}}{\lambda_{iob}}(T_{iob} - T_0) = \frac{\alpha_{ia}l_{ia}}{\lambda_{ia}}(T_{ia} - T_{is}), \tag{9}$$

denoting preservation of the generalized Biot number, (7) and (8) will have identical form and the temperatures at corresponding points of the object and model will be equal:

$$T_i \equiv T_{ia} = T_{iob}.$$

If the emissivity of the shield $\varepsilon_{is} = 1$

$$\alpha_{ia} = \varepsilon_{ia}\sigma f(T_i T_{is}). \tag{10}$$

Here σ is the Stefan–Boltzmann constant, and $f(T_i T_{is}) = T_i^3 + T_i^2 T_{is} + T_i T_{is}^2 + T_{is}^3$.

For the object

$$\alpha_{iob} = \varepsilon_{iob}\sigma f(T_i T_0), \tag{11}$$

where $f(T_i T_0)$ is analogous to $f(T_i T_{is})$.

Substituting (10) and (11) into (9) and taking account of the fact that $l_{ia}/l_{iob} = c$ we obtain

$$c\,\frac{\varepsilon_{ia}\sigma(T_i - T_{is})f(T_i T_{is})}{\lambda_{ia}} = \frac{\varepsilon_{iob}\sigma(T_i - T_0)f(T_i T_0)}{\lambda_{iob}}.$$

Setting $\varepsilon_{ia} = \varepsilon_{iob}$ and $\lambda_{ia} = \lambda_{iob}$, i.e., the model is made of the same materials as the object, we have

$$\frac{T_i^4 - T_0^4}{T_i^4 - T_{is}^4} = c. \tag{12}$$

By choosing the temperature of the corresponding regions of the shield so that (12) is satisfied we can obtain a temperature distribution in the model similar to that in the object. To obtain identical temperatures at corresponding points the power dissipated in the model must be increased by a factor c.

NOTATION

ε is the surface emissivity;
λ is the thermal conductivity;

α	is the radiative heat-transfer coefficient;
l	is a characteristic linear dimension;
c	is the length scale for the model;
P	is the power;
T_{iob}, T_{ia}, T_{is}, T_0	are the respectively the temperatures of the object, the model, the shield, and the environment;
σ	is the Stefan–Boltzmann constant.

LITERATURE CITED

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